Aerodynamics of the Darrieus Rotor

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Introduction

RENEWED interest in the wind as an alternative energy source has resulted in a number of investigations of unconventional wind-powered machines. South and Rangi[1] have developed a vertical axis turbine of a type earlier proposed by Darrieus.[2] This device is illustrated in Figure 1a. The aerodynamics of Darrieus-type rotors has been examined both analytically[3-5] and experimentally.[6,7]

This Note compares published experimental data on the Darrieus Rotor with the flow model of Wilson and Lissaman.

Theory

To analyze a Darrieus-type crosswind-axis device we adopt the standard approach of wing theory, which is to express the forces on the system by a momentum analysis of the wake as well as by an airfoil theory at the lifting surface itself. The expression for these forces contains unknown induced flows. By equating the wake and wing forces one obtains sufficient equations to determine the induced flows.

For the device considered, we assume that each spanwise station (parallel to the axis of rotation) behaves independently in the sense that the forces on the device at each station may be equated to the wake forces. In general, these devices can experience a windwise as well as a cross-wind force, so that the wake can be deflected to the side.

Using the same assumption as in vortex theory of propellers, we will assume the induced flows at the device are one half their value in the wake. Thus, we obtain that if the wake windwise perturbation is $\Delta V \simeq -2aV_\infty$, then at the device itself the incoming flow has velocity $V_a = V_\infty(1\text{-}a)$, giving the flow system illustrated in Figure 1b.

We can equate the force on the airfoil to the change in momentum in the streamtube which the airfoil occupies. Let the streamtube be of width dx when the airfoil goes from angular position θ to position $\theta + \mathrm{d}\theta$. The width d$x$ is related to dθ by

$$\mathrm{d}x = R\mathrm{d}\theta\ |\sin\theta| \qquad (1)$$

The process will repeat itself every revolution so the time interval of our analysis shall be one period which is $2\pi/\Omega$. Of this time period, the airfoil will spend a time increment of $\mathrm{d}\theta/\Omega$ in the front portion of the streamtube and another time increment of $\mathrm{d}\theta/\Omega$ in the rear portion of the wake. Since the streamwise force contribution is assumed to be symmetrical with respect to the angles $\pm\theta$ we may write the blade force equation for the time period $2\pi/\Omega$ as

$$(\hat{j}\cdot(\mathrm{d}\hat{F}/\mathrm{d}z))_{\text{blade}} = -1/2\rho W_e Nc\, C_L\, V_t \sin\theta\ (2\mathrm{d}\theta/\Omega) \qquad (2)$$

where N is the number of blades and

$$W_e^2 = V_a^2\ (1\text{-}\sin^2\theta\sin^2\gamma) + V_t^2 + 2V_a V_t\cos\theta \qquad (3)$$

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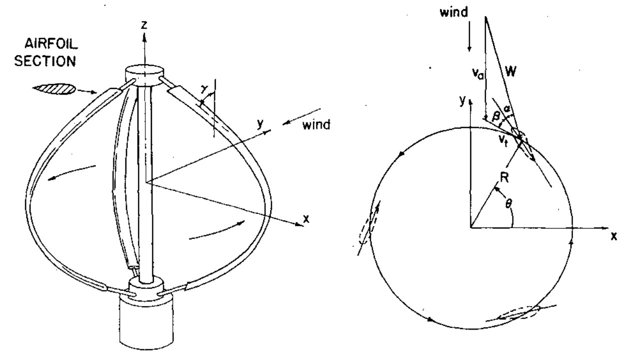

Fig. 1 Darrieus wind turbine and flow system.

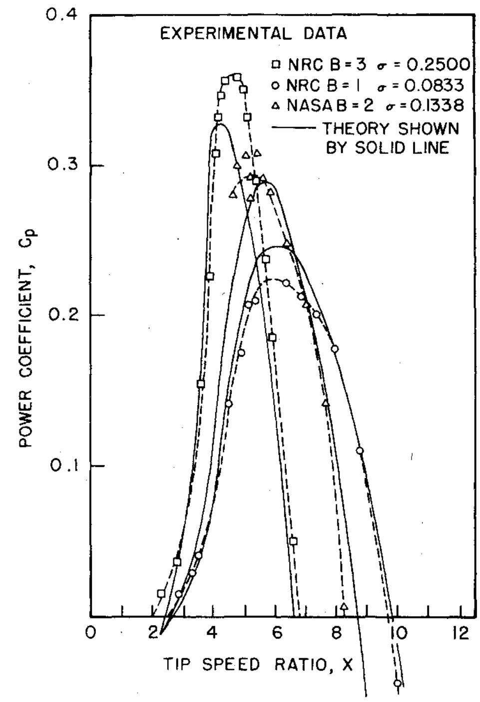

Fig. 2 Comparison of analytical and experimental power coefficients for a Darrieus rotor. $X = R_M\Omega/V_\infty$, $\sigma = Nc/2R_M$.

and

$$V_t = R\Omega,\ C_L = C_L(\alpha), \tan\alpha = V_a\sin\theta\cos\gamma/(V_a\cos\theta + V_t)$$

Now the momentum equation yields the force in the streamtube as

$$(\hat{j}\cdot(\mathrm{d}\hat{F}/\mathrm{d}z))_{\text{momentum}} = -\rho R d\theta\ |\sin\theta|\ (1\text{-}a)\,V_\infty 2V_\infty a\,(2\pi/\Omega) \qquad (4)$$

Equating the forces determined from momentum analysis with the blade generated forces one obtains

$$a(1\text{-}a) = \frac{Nc}{4\pi R}\ \frac{R\Omega}{V_\infty}\ C_L\ \frac{W_e}{V_\infty}\ \sin\theta/|\sin\theta| \qquad (5)$$

which can be solved for a by iteration. After solving the preceding equation, the local velocities are known so that the local blade forces may be calculated and resolved into torque, streamwise, and side force components. Aerodynamic drag can be determined from local flow conditions and subtracted from the lift-produced torque.

Comparison with Experiment

Studies of Ref. 6 and 7 give experimental results for Darrieus rotors with diameters of 12 to 14 ft with one, two, or

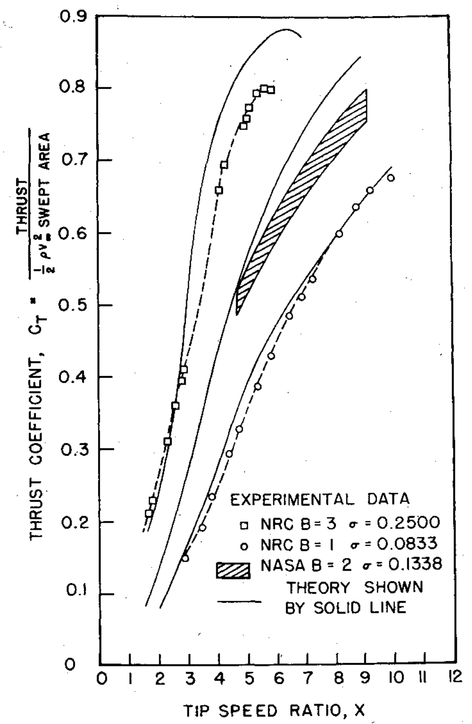

Fig. 3 Comparison of analytical and experimental thrust coefficients for a Darrieus rotor.

three blades. The blades used in these studies were NACA 0012 sections having about a 6-in. chord. The resulting Reynolds numbers for the blades were of the order of $3 \cdot 10^5$. References 6 and 7 were used to check the analytical formulations of Templin, Muraca, and Wilson and Lissaman. The later analytical formulation was found to give the best results when compared to the data. Figures 2 and 3 present a performance comparison between analytical and experimental results. There still exist notable differences between theory and experiment.

Conclusions

Several conclusions concerning the analysis are to be made.

1) For perfect blade alignment ($\beta = 0$), the average side force is zero. If the angle β is not zero, a net force perpendicular to the wind will be developed.

2) The reduced frequency ($\Omega c/2V_{local}$) has the same value for all test data used. The value is about 0.04, small enough that the aerodynamics should be quasisteady unless static stall is approached by the blades. Incorporation of the effects of unsteady lift into the analysis will result in two effects. First, the magnitude of the lift developed will be reduced. This will result in lower predicted performance. Second, the lift will lag the angle of attack. The principle effect of the phase lag is that the rotor will experience a net side force.

3) Available test data covers power and overall force measurements only. A wake velocity survey has not been made by any of the investigators (Muraca[7] made one traverse). Since any aerodynamic theory for the Darrieus rotor requires explicit knowledge of the induced velocity, a fundamental piece of information has yet to be obtained.

4) Wind tunnel tests to date have been made at low blade chord Reynolds numbers. A larger Darrieus rotor (diameter of the order of 100 ft) is expected to have chord Reynolds numbers on the order of 10^6. This increase in Reyonlds number is expected to yield lower sectional drag coefficients and higher sectional lift coefficients. Both of these effects will increase the performance of the Darrieus rotor by a significant amount.

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Announcement: 1976 Author and Subject Index

The indexes of the four AIAA archive journals (*AIAA Journal, Journal of Spacecraft and Rockets, Journal of Aircraft,* and *Journal of Hydronautics*) will be combined and mailed separately early in 1977. In addition, papers appearing in volumes of the *Progress in Astronautics and Aeronautics* book series published in 1976 will be included. Librarians will receive one copy of the index for each subscription which they have. Any AIAA member who subscribes to one or more Journals will receive one index. Additional copies may be purchased by anyone, at $10 per copy, from the Circulation Department, AIAA, Room 730, 1290 Avenue of the Americas, New York, New York 10019. **Remittance must accompany the order.**

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